LESSON 5.3

NAME ______________________ DATE ________

Reteaching with Practice

For use with pages 279–285

GOAL **Use properties of medians of a triangle and use properties of altitudes of a triangle**

VOCABULARY

A **median of a triangle** is a segment whose endpoints are a vertex of the triangle and the midpoint of the opposite side.

The point of concurrency of the three medians of a triangle is called the **centroid of the triangle.**

An **altitude of a triangle** is the perpendicular segment from a vertex to the opposite side or to the line that contains the opposite side.

The lines containing the three altitudes are concurrent and intersect at a point called the **orthocenter of the triangle.**

Theorem 5.7 Concurrency of Medians of a Triangle
The medians of a triangle are concurrent at a point that is two thirds of the distance from each vertex to the midpoint of the opposite side.

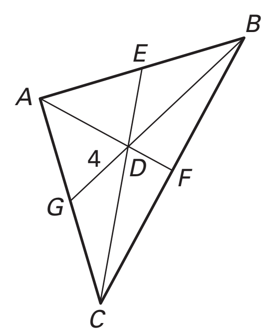
Theorem 5.8 Concurrency of Altitudes of a Triangle
The lines containing the altitudes of a triangle are concurrent.

EXAMPLE 1 *Using the Medians of a Triangle*

D is the centroid of $\triangle ABC$ and $DG = 4$. Find the indicated values.

a. Find BG.

b. Find BD.



SOLUTION

a. Because D is the centroid of $\triangle ABC$, $BD = \frac{2}{3}BG$. Then

$DG = BG - BD = \frac{1}{3}BG$. Substituting 4 for DG, $4 = \frac{1}{3}BG$, so $BG = 12$.

b. $BD = \frac{2}{3}BG$, so by substituting 12 for BG, you get $BD = \frac{2}{3}(12) = 8$. So, $BD = 8$.

Exercises for Example 1

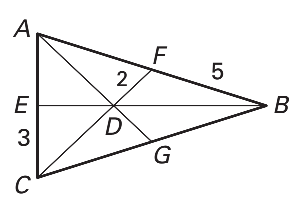
Use the figure and the given information. *D* is the centroid of $\triangle ABC$, $\overline{BE} \perp \overline{AC}$, $\overline{AB} \cong \overline{CB}$, $FB = 5$, $EC = 3$, and $DF = 2$.

1. Find CF.

2. Find CG.

3. Find CD.

4. Find the perimeter of $\triangle ABC$.



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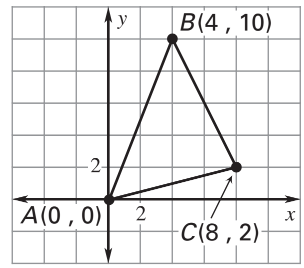
EXAMPLE 2 Finding the Centroid of a Triangle

Find the coordinates of the centroid of $\triangle ABC$.



SOLUTION

You know that the centroid is two thirds of the distance from each vertex to the midpoint of the opposite side. If you find the midpoint of any side and draw a segment from that point to the opposite vertex, you will have a segment which contains the centroid. You can then find the length of the segment. Finally, you know that the centroid is two thirds of the length of this segment from the vertex.

Find the midpoint of $\overline{AC}$. The midpoint of $\overline{AC}$ is $\left(\frac{0+8}{2}, \frac{0+2}{2}\right) = (4, 1)$. A median can be drawn from this midpoint to the vertex B. Use the distance formula to find the length of this median.

$$d = \sqrt{(4-4)^2 + (10-1)^2} = \sqrt{0^2 + 9^2} = 9$$

By Theorem 5.7, the centroid is two thirds of this distance, or $\frac{2}{3} \cdot 9 = 6$ units, down from B along the median. The coordinates of the centroid are therefore $(4, 10 - 6)$, or $(4, 4)$.

Exercises for Example 2

Find the coordinates of the centroid of the triangle with the given vertices.

5. $A(0, 0)$, $B(10, 0)$, $C(5, 6)$

6. $D(-5, 2)$, $E(-3, 6)$, $F(-7, 10)$

7. $G(-1, 2)$, $H(-3, 10)$, $I(10, 6)$ (*Hint:* Find the median from I to $\overline{GH}$.)